Exercise 57

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(t) = 2\cos t + \sin 2t, \quad [0, \pi/2]$$

Solution

Take the derivative of the function.

$$\begin{aligned} f'(t) &= \frac{d}{dt}(2\cos t + \sin 2t) \\ &= 2\frac{d}{dt}(\cos t) + \frac{d}{dt}(\sin 2t) \\ &= 2(-\sin t) + \cos 2t \cdot \frac{d}{dt}(2t) \\ &= -2\sin t + \cos 2t \cdot (2) \\ &= -2\sin t + (1 - 2\sin^2 t) \cdot (2) \\ &= -2(2\sin^2 t + \sin t - 1) \end{aligned}$$

Set $f'(t) = 0$ and solve for t.

$$-2(2\sin^2 t + \sin t - 1) = 0$$

$$2\sin^2 t + \sin t - 1 = 0$$

$$\sin t = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$$

$$\sin t = \frac{-1 \pm \sqrt{9}}{4}$$

$$\sin t = \frac{1}{2} \quad \text{or} \quad \sin t = -1$$

$$t = \frac{\pi}{6} + 2\pi n \quad \text{or} \quad t = \frac{5\pi}{6} + 2\pi n \quad \text{or} \quad t = \frac{3\pi}{2} + 2\pi n, \quad n = 0, \pm 1, \pm 2, \ldots$$

Only $t = \pi/6$ is within $[0, \pi/2]$, so evaluate f here.

$$f\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{2\pi}{6}\right) = \frac{3\sqrt{3}}{2} \approx 2.59808 \qquad \text{(absolute maximum)}$$

Now evaluate the function at the endpoints of the interval.

$$f(0) = 2\cos 0 + \sin 0 = 2$$

$$f\left(\frac{\pi}{2}\right) = 2\cos\frac{\pi}{2} + \sin \pi = 0 \qquad \text{(absolute minimum)}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval The graph of the function below illustrates these results.